Summary - Mathematics

This appendix in mathematics is intended as a brief review of o0perations and methods. One should be totally familiar with basic algebraic techniques, analytic geometry and trigonometry. The section containing differential and integral calculus is more detailed and is intended for those students who have difficulty applying calculus concepts to physical situations.

Scientific Notation:

Many quantities that scientists deal with often have very large or very small values. For example, the speed of light is about 300 000 000 m/s and the ink required to make a dot over an *i* in this print has a mass of about 0.000 000 000 08 kg. Obviously, it is very cumbersome to read, write and keep track of numbers such as these. One can avoid this problem by using a method dealing with powers of the number 10. The number of zeros correspond to the power to which 10 is raised, called the exponent of 10. In the case the number of places the decimal point is to the left of the digit 1 equals the value of the (negative) exponent. Numbers expressed as some power of 10 multiplied by another number between 1 and 10 are said to be in scientific notation. For example, the speed of light $3{\cdot}E^{8}$m/s or the mass of the inc droplet $80{\cdot}E^{-9}$kg.

Value	Power		Prefix	Symbol
1 000 000 000 000 000 000	10^{18}	$1{\cdot}E^{18}$	Exa	E
1 000 000 000 000 000	10^{15}	$1{\cdot}E^{15}$	Peta	P
1 000 000 000 000	10^{12}	$1{\cdot}E^{12}$	Tera	T
1 000 000 000	10^{9}	$1{\cdot}E^{9}$	Giga	G
1 000 000	10^{6}	$1{\cdot}E^{6}$	Mega	M
1 000	10^{3}	$1{\cdot}E^{3}$	Kilo	k
100	10^{2}	$1{\cdot}E^{2}$	Hecto	h
10	10^{1}	$1{\cdot}E^{1}$	Deca	da
1	1	1		
0.1	10^{-1}	$1{\cdot}E^{-1}$	deci	d
0.01	10^{-2}	$1{\cdot}E^{-2}$	centi	c
0.001	10^{-3}	$1{\cdot}E^{-3}$	milli	m
0.000 001	10^{-6}	$1{\cdot}E^{-6}$	micro	µ
0.000 000 001	10^{-9}	$1{\cdot}E^{-9}$	nano	n
0.000 000 000 001	10^{-12}	$1{\cdot}E^{-12}$	pico	p
0.000 000 000 000 001	10^{-15}	$1{\cdot}E^{-15}$	femto	f
0.000 000 000 000 000 001	10^{-18}	$1{\cdot}E^{-18}$	atto	a

When numbers expressed in scientific notation are being multiplied or divided, the following general rule is very useful (see also rules of exponents, p.3); where *n* and *m* can be *any* numbers (not necessarily integers).

$$10^{n} \cdot 10^{m} = 10^{n+m}$$
$$10^{n}/10^{m} = 10^{n} \cdot 10^{-m} = 10^{n-m}$$

For example, $1{\cdot}E^{2} \cdot 1{\cdot}E^{5} = 1{\cdot}E^{7}$. The rule also applies if one of the exponents is negative.
For example, $1{\cdot}E^{3} \cdot 1{\cdot}E^{-8} = 1{\cdot}E^{-5}$.

Algebra:

When algebraic operations are performed, the laws of arithmetic apply. Symbols such as x, y, and *z* are usually used to represent quantities that are not specified, what are called the unknowns. Before going into any further details briefly the most commonly used notations in algebraic expressions:

Abbreviations: Mathematical symbols

=	is equal to
≠	is not equal to
≈	is approximately equal to
∝	is proportional to
>	is greater than
≥	is greater than or equal to
>>	is much greater than
<	is less than
≥	is less than or equal to
<<	is much less than
Δx	change in x
│x│	absolute value of x
n!	n·(n-1)·(n-2)·....·1
∑	sum
lim	limit
Δt→0	Δt approaches zero
dx/dt	derivative of x with respect to t
∂x/∂t	partial derivative of x with respect to t
∫	integral

Now, consider the simple equation $8 \cdot x = 32$

If we wish to solve for x, we can divide (or multiply) each side of the equation by the same factor without destroying the equality. In this case,

if we divide both sides by 8, we have: $8 \cdot x/8 = 32/8$

Which gives us the final result: $x = 4$

Next consider the equation $x + 2 = 8$

In this type of expression, we can add or subtract the same quantity from each side. If we subtract 2 from each side, we get: $x + 2 - 2 = 8 - 2$

Which gives us again the final result: $x = 6$

In general, if x + a = b, then x = b - a.

Now consider the equation $x/5 = 9$

If we multiply each side by 5,

we are left with x on the left by itself and 45 on the right: $5 \cdot x/5 = 5 \cdot 9$

we obtain the final result: $x = 45$

In all cases, *whatever operation is performed on the left side of the equality must also be performed on the right side*. The following rules for multiplying, dividing, adding, and subtracting fractions should be recalled, where a, b, and c are three numbers:

	Rule
Multiplying	$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$
Dividing	$\frac{a/b}{c/d} = \frac{a \cdot d}{c \cdot b}$
Adding	$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}$

General Rules:

1. The same quantity can be added to or subtracted from each side of the equation.
2. Each side of the equation can be multiplied or divided by the same quantity.
3. Each side of the equation can be raised to the same power.

It is important to understand that the preceding rules apply to each *side* of the equation and not the each *term* in the equation.

Direct and Inverse Proportion: The relationship of direct and inverse proportion are frequently used in any scientific discipline;

Direct P.: Any mathematical expression in which the operands are multiplicatively related to each other; e.g.: $c = a \cdot b$

Inverse P.: Any mathematical expression in which the operands are inversely related to each other; e.g.: $c = a/b$

Solving Simultaneous Linear Equations: Consider an equation such as 3·x + 5·y = 15, which has two unknowns, x and y. Such an equation does not have a unique solution. That is, (x = 0, y = 3), (x = 5, y = 0), and (x = 2, y = 9/5) are all solutions to this equation. If a problem has two unknowns, a unique solution is possible only if we have *two* equations. In general, if a problem has *n* unknowns, its solution requires *n* independent equations. In order to solve two simultaneous equations involving two unknowns, x and y, we solve one of the equations for x in terms of y and substitute this expression into the other equation.

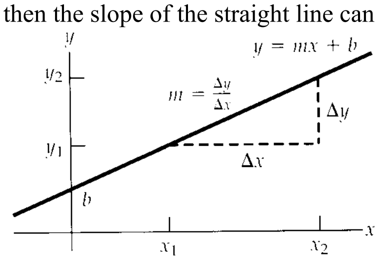
To solve the following simultaneous equations: EQ-1: 5·x + y = -8
EQ-2: 2·x - 2y = 4

From EQ-2, substitution of this into EQ-1 gives 5·(y + 2) + y = -8
Alternatively, multiply each term in EQ-1 by the y = 3, x = -1
factor 2 and add the result to EQ-2: l0·x + 2·y= -16, solution as above;

Two linear equations with two unknowns can also be solved by a graphical method. If the straight lines corresponding to the two equations are plotted in a conventional coordinate system, the intersection of the two lines represents the solution. For example, consider the two equations. The intersection of the two lines has the coordinates x = 5, y =3. This represents the solution to the equations. You should check this solution by the analytical technique discussed above.

Function (or Equation, yielding a unique solution): The ordinate *y* of a point of a curve is uniquely associated with an abscissa *x*. Hence, the curve defines a function, which is denoted by the expression $y = f(x)$, with the resulting curve called a graph. For each *x* (independent variable) there is an associated and unique *y* (dependent variable).

- **Linear** F.: A function in which the variables occur only to the **first** power is said to be linear.[1]
  A linear function relating *y* and *x* can always be put into the standard form: $y = m \cdot x + b$,
  where *m* and *b* are constants;
  This equation is referred to as being linear because the graph of y versus x is a straight line, as shown in figure below. The constant *b,* called the intercept, represents the value of y at which the straight line intersects they axis. The constant *m* is equal to the slope of the straight line and is also equal to the tangent of the angle that the line makes with the *x*-axis.
  If any two points on the straight line are specified by the coordinates (x_1, y_1) and (x_2, y_2), as in figure, then the slope of the straight line can be expressed as:



graph of the linear equation $y = m \cdot x + b$,
where *b* is the intercept and $m = \Delta y/\Delta x = tan\theta$

$$m=(y_2-y_1)\cdot(x_2-x_1)=\frac{\Delta y}{\Delta x}=tan\theta$$

for *b* = 0, the equation becomes
$y = m \cdot x$, the graph runs through the origin

Note that *m* and *b* can have either positive or negative values. If m > 0, the straight line has a *positive* slope, as shown above. If a < 0, the straight line has a negative slope.

- **Power** F. (Equations of n^th^-degree): The exponent is a constant; the independent variable is the base of a power; (not to be confused with the e-function!). The quantity *n* is called **power**, or the **exponent**, of *x*;
  e.g.: $y = a \cdot x^4 = a \cdot (x \cdot x \cdot x \cdot x)$; if *n* = 4
  a and *n* being constants with $a \neq 0$; $y = a \cdot x^n$
  if *x* = 0, the entire equation *y* = 0; $y = a_n \cdot x^n + a_{(n-1)} \cdot x^{(n-1)} + ... + a_0$

  General Rules:

[1] http://en.wikipedia.org/wiki/Linear_equation

When powers of a given quantity x are multiplied, the following rule applies: e.g.: $x^2 \cdot x^4 = x^{2+4} = x^6$ — $x^n \cdot x^m = x^{n+m}$

When dividing the powers of a given quantity, e.g.: $x^8/x^2 = x^{8-2} = x^6$ — $x^n/x^m = x^{n-m}$

A power that is a fraction, such as 1/n, — $x^{1/n} = \sqrt[n]{x}$ (the "nth" root of x)

corresponds to a root as follows: — $x^{1/n} = \sqrt[n]{x}$

e.g.: $4^{1/3} = \sqrt[3]{4} = 1.5874$. — $x^0 = 1$

other rules of exponents — $x^1 = x$

$$\left(x^n\right)^m = x^{n \cdot m}$$

Quadratic F.: is a function of 2nd degree or quadratic function; a special quadratic function; it is a quadratic *parabola* with the vertex at the origin. $y = x^2$,

The general quadratic function is a compound of a special quadratic combined with a linear function.[2] Its generalized form is shown on the right;

here, x is the unknown quantity and a, b, and c are numerical coefficients referred to as coefficients of the equation.

$$y = a \cdot x^2 + b \cdot x + c$$

c, +/- shift along the y-axis

$a \neq 0$, for a quadratic EQ.

This equation has two roots (solutions), given by:

${}_1x_2$ being the two solutions x_1 and x_2 with

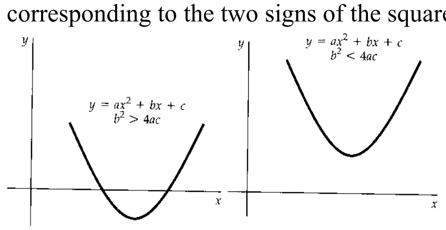
$(b^2 - 4 \cdot a \cdot c) \geq 0$, condition for real solutions.

$${}_1x_2 = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

The results ${}_1x_2$ is denoted in such a way that x+ refers to the root corresponding to the positive sign and x- refers to the root corresponding to the negative sign.

The equation $x^2 + 5 \cdot x + 4 = 0$ has the following roots corresponding to the two signs of the square-root term:

$${}_1x_2 = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} = \frac{-5 \pm 3}{2} = -1 \ \& \ -4$$



- graph of y versus x when $y = a \cdot x^2 + b \cdot x + c$ for the case $b^2 > 4 \cdot a \cdot c$; the two values of x for which $y = 0$ satisfy the quadratic equation;
- when $b^2 = 4 \cdot a \cdot c$, the graph of y versus x is tangent to the x-axis at the point $x = -b/(2 \cdot a)$
- graph of y versus x when $y = a \cdot x^2 + b \cdot x + c$ for the case $b^2 < 4 \cdot a \cdot c$; in this case, there are no (real) values for x for which $y = 0$;

Factoring: Equations can often be simplified by factoring:

1. Common factor: $2ax + 3ay = a\,(2x + 3y)$
2. Perfect square: $x^2 \pm 2xy + y^2 = (x \pm y)^2$
3. Difference of square: $x^2 - y^2 = (x + y) \cdot (x - y)$

Binomial Expansion: The binomial theorem is very useful for making approximations. If n is a positive integer, there are n+1 terms in series. If n is a real number other than a positive integer, there are an infinite number of terms. The series is valid for any value of n if x^2 is less than 1. It is also valid for $x^2 = 1$ if n is positive. The series is particularly useful if $|x|$ is much less than1. Then each term is much smaller than the precious term and we can drop all but the first 2 or3 terms.

with n being a constant

$$(1+x)^n = 1 + n \cdot x + \frac{n(n-1)}{2!} \cdot x^2 + \frac{n(n-1)(n-2)}{3!} \cdot x^3 + \frac{n(n-1)(n-2)(n-3)}{4!} \cdot x4 + \ldots.$$

series expansion in general form:

$$(a+b)^n = a^n + \frac{n}{1!} \cdot a^{n-1} \cdot b + \frac{n(n-1)}{2!} \cdot a^{n-2} \cdot b^2 + \frac{n(n-2)}{3!} \cdot a^{n-3} \cdot b^3 + \ldots.$$

[2] http://en.wikipedia.org/wiki/Quadratic_equation

- **Exponential** F.: A function characterized by an independent variable in the exponent (not to be confused with a power function), i.e.: the rate of change of a quantity is proportional to the quantity itself; the quantity decreases or increases exponentially. Hence, the *e- function* with the variable exponent, can be + or - (rising or falling graph); the dimension of the exponent has to be **1** (dimensionless!).

  $a \neq 0$, $a > 1$, $q > 0$ — $y = q \cdot a^x$

  e, Eulers number 2.71828 — $y = q \cdot e^x$

  some basic rules when working with exponents (analog to those displayed in the section of the the power function):

  $$y = a^x \cdot a^x = a^{x+x} = a^{2x}$$

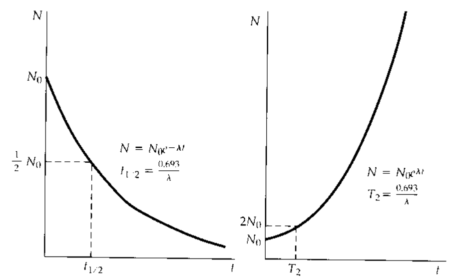
  $$y = \left(a^x\right)^x = a^{x \cdot x}$$

  $$y = \frac{1}{a^x} = a^{-x}$$

  $$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$$

  an example of *exponential* de/in-crease is nuclear decay: $N_{(t)} = N_0 \cdot e^{-\lambda \cdot t}$

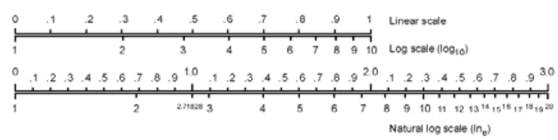
  N_0 as the number of radioactive nuclei at t=0:
  - graph of N versus t when N decreases exponentially; the time $t_{1/2}$ is the time it takes for N to decrease by one-half;
  - graph of N versus t when N increases exponentially; the time T_2 is the time it takes for N to double;



- **Logarithmic** F.:[3] The logarithm of x with respect to the base a is equal to the exponent to which the base must be raise in order to satisfy the expression $x = a^y$; — $y = log_a(x)$

  Inverse function (antilogarithm) of the exponential function: — $x = antilog_a(y)$

  the domain of the log function is the interval x>0; that is log's can only be taken from positive numbers; the range of the log can take every real number a>1, usually based on the value of 10. an illustration of the *lin-log* relationship is given at the right; i.e. the result of $log_{10}(5)$ can be graphically solved to be approx. 0.7; while $log_{10}(200) = log(2) + 2 \cdot log(10) \approx 0.3 + 2 \approx 2.3$.



  In practice, the two bases most often used are base 10, called the common logarithm base (lg) and base e = 2.718, called the natural logarithm base (ln). e, Euler's number corresponds to 2.71828

  $$log_{10}(10) = 1 = ln(e)$$

  $$log_{10}(1) = 0 = ln(1)$$

  $$y = log_e(x) = ln(x)$$

  $$ln_e(x) = 2.302585 \cdot log_{10}(x)$$

  $$-y = \ln(x) - \ln(y) \leftrightarrow y = \ln(y) - \ln(x)$$

  $$ln(1 \pm x) = \pm x - \frac{1}{2}x \pm \frac{1}{3}x^2 - \frac{1}{4!}x^3 \pm ....$$

  General Rules and useful properties of log's:

  $$y = \lg(x) \leftrightarrow x = 10^y$$

  $$y = \ln(x) \leftrightarrow x = e^y$$

  $$y = \ln(e) = 1 \leftrightarrow x = e^0 = 1$$

  $$y = \ln(e^x) = x \leftrightarrow x = e^{\ln y} = y$$

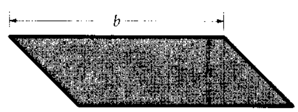
  Some basic rules when working with logarithms:

$x, y > 0$	$\log(x) + \log(y)$ = $\log(x \cdot y)$	
$x, y > 0$	$\log(x) - \log(y)$ = $\log(x/y)$	
$x > 0$	$\log(1/x)$ = $log(x^{-1}) = -\log(x)$	
$x, y > 0$	$a \cdot \log(x)$ = $\log(x^a)$	
$x > 0$; n, any real number ($\in$ R)	$\frac{1}{n} \cdot log(x)$ = $log x^{\frac{1}{n}} = log \sqrt[n]{x}$	

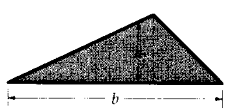
[3] http://en.wikipedia.org/wiki/Logarithm

Geometry: The ratio of the circumference of a circle to its diameter is a natural number π, which has the

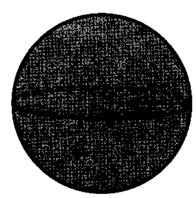
approximate value	$\pi = 3.141592$	
circumference of a circle	$C = \pi \cdot d$	*d*, diameter
area of a circle	$A = \pi \cdot r^2$	*r*, radius
spherical surface area	$A = 4 \cdot \pi \cdot r^2$	
spherical volume	$V = 4 \cdot \pi \cdot \frac{r^3}{3}$	
cylindrical surface	$A = 2 \cdot \pi \cdot r \cdot L$	*L*, length of cylinder
cylindrical volume	$V = \pi \cdot r^2 \cdot L$	
circumference of a rectangle	$C = 2 \cdot a \cdot b$	*a, b*, legs of the rectangular
rectangular area	$A = a \cdot b$	
area of parallelogram	$A = b \cdot h$	*h*, height of parallelogram
circumference of trapeze	$C = a + b + c + d$	*a, b, c, d*, legs of the trapeze
trapezial area	$A = (a + c) \cdot \frac{h}{2}$	*a, c*, the parallel legs perpendicular to h, height
circumference of a triangle	$C = a + b + c$	*a, b, c*, legs of the triangle
triangular area	$A = b \cdot \frac{h}{2}$	*h*, height of triangle



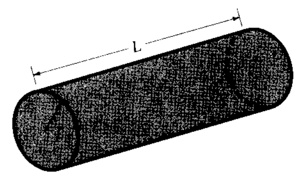
Area of parallelogram
$A = bh$



Area of triangle
$A = \frac{1}{2} bh$



Spherical surface area
$A = 4\pi r^2$
Spherical volume
$V = \frac{4}{3}\pi r^3$



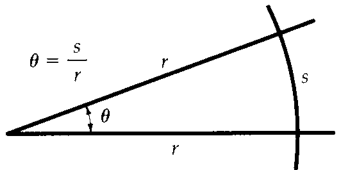
Cylindrical surface area
$A = 2\pi rL$
Cylindrical volume
$V = \pi r^2 L$

formula of cylindrical surface area does not include the end faces

Trigonometry: The angle between two intersecting straight lines corresponds to the center at the intersection of the lines with the circular arc divided into 360 parts (called **degrees**). The number of degrees in the arc between the lines is measured of angle between the lines. For very small angles, the degree is divided into minutes (') and seconds (") with 1' = 1°/60 and 1" = 1'/60 = 1°/3600.

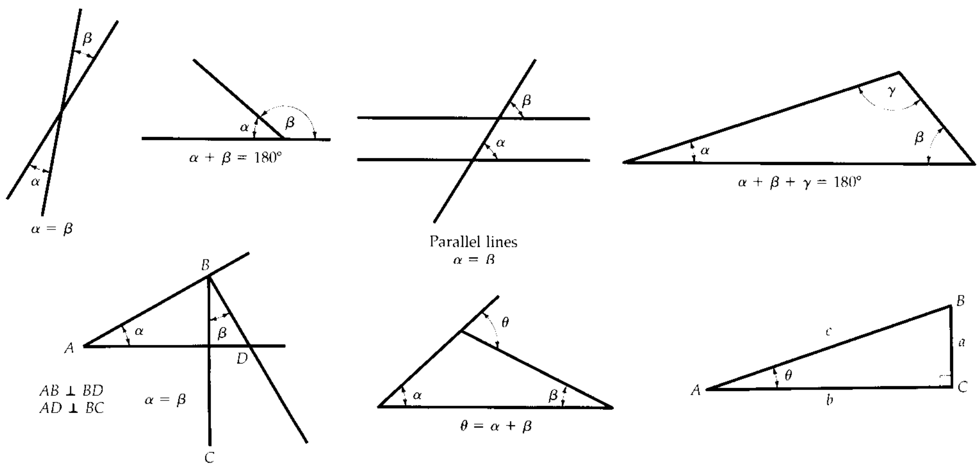
For scientific work, a more useful measure of an angle is the radian (rad), which is defined as the length of the circular arc between the lines divided by the radius of the circle.

$\theta = s/r$

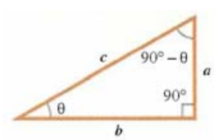


s is the arc length
r is the radius of the circle,
the angle θ is measured in radians;
360° = 2·π·rad;
1° = 2·π/360
1rad = 360°/(2·π) = 57.3°

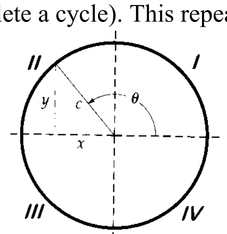
some useful relations for angles:
bottom right: right triangle w/ sides of length *a* and *b* and a hypotenuse of length *c.*



The relationships of the right triangle are:
a = opposite side
b = adjacent side
c = hypothenuse



Trigonometric F.: Periodical function with based on a repeating pattern, the period (an interval required to complete a cycle). This repeating pattern oscillates within a minimum and a maximum elongation.



Quadrant	Interval	cosine / sine
I	0° < θ < 90°	+ / +
II	90° < θ < 180°	- / +
III	180° < θ < 270°	- / -
IV	270° < θ < 360°	+ / -

$\cos(-\theta) = \cos\theta$
$\sin(-\theta) = -\sin\theta$

$$\sin\theta = \frac{y}{c} = \frac{\text{side opposite }\theta}{\text{hypotenuse}}$$

y, leg adjacent to θ

$$\cos\theta = \frac{x}{c} = \frac{\text{side adjacent to }\theta}{\text{hypothenuse}}$$

x, leg opposite to θ

$$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta} = \frac{\text{side opposite }\theta}{\text{side adjacent to }\theta}$$

c, hypotenuse

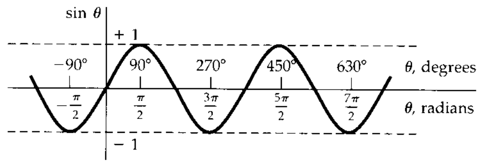
$$ctan\theta = \frac{x}{y} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

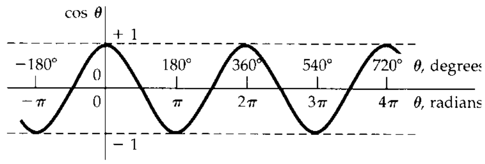
$$\theta = \arcsin\frac{y}{c} = \sin^{-1}\frac{y}{c}$$

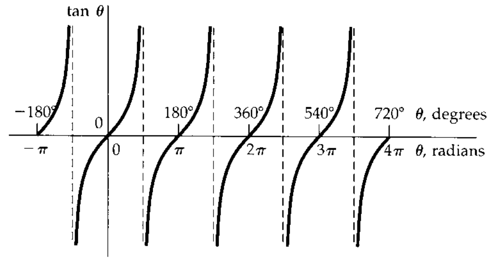
simplified notation

$$\theta = \arccos\frac{x}{c} = \cos^{-1}\frac{x}{c}$$

$$\theta = \arctan\frac{y}{x} = tang^{-1}\frac{y}{x}$$







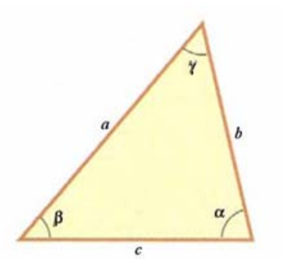
T. **Relations**:

$sin(-\theta) = -sin\theta$
$cos(-\theta) = -cos\theta$
$tan(-\theta) = -tan\theta$

application of the Pythagorean theorem:
$c^2 = a^2 + b^2$

Law of cosine (applies to any triangle)
$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos\alpha$
$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot cos\beta$
$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot cos\gamma$

Law of sine (applies to any triangle)
$$\frac{\alpha}{sin\alpha} = \frac{b}{sin\beta} = \frac{c}{sin\gamma}$$



Some other trigonometric identities

$cos^2\theta + sin^2\theta = 1$
$cos(2\cdot\theta) = cos^2\theta - sin^2\theta$
$sin(2\cdot\theta) = 2 \cdot sin\theta \cdot cos\theta$

$$1 - cos\theta = 2 \cdot sin^2\frac{\theta}{2}$$

$$sin^2\frac{\theta}{2} = \frac{1}{2}(1 - cos\theta) \qquad cos^2\frac{\theta}{2} = \frac{1}{2}(1 + cos\theta)$$

$$tan(2\cdot\theta) = \frac{2 \cdot tan\theta}{1 - tan^2\theta} \qquad tan\frac{\theta}{2} = \sqrt{\frac{1 - cos\theta}{1 + cos\theta}}$$

$$sin(\alpha \pm \beta) = sin\alpha \cdot cos\beta \mp coss\alpha \cdot sin\beta \qquad cos(\alpha \pm \beta) = cos\alpha \cdot cos\beta \mp sin\alpha \cdot sin\beta$$

$$sin\varphi + sin\theta = 2 \cdot sin\frac{\varphi + \theta}{2} \cdot cos\frac{\varphi - \theta}{2} \qquad sin\varphi - sin\theta = 2 \cdot cos\frac{\varphi + \theta}{2} \cdot sin\frac{\varphi - \theta}{2}$$

$$cos\varphi + cos\theta = 2 \cdot cos\frac{\varphi + \theta}{2} \cdot cos\frac{\varphi - \theta}{2} \qquad cos\varphi - cos\theta = -2 \cdot sin\frac{\varphi + \theta}{2} \cdot sin\frac{\varphi - \theta}{2}$$

$$sin\alpha \pm sin\beta = 2 \cdot sin\left(\frac{1}{2} \cdot (a \pm b)\right) \cdot cos\left(\frac{1}{2} \cdot (a \mp b)\right)$$

$$cos\alpha + cos\beta = 2 \cdot cos\left(\frac{1}{2} \cdot (a + b)\right) \cdot cos\left(\frac{1}{2} \cdot (a - b)\right) \qquad cos\alpha - cos\beta = 2 \cdot sin\left(\frac{1}{2} \cdot (a + b)\right) \cdot sin\left(\frac{1}{2} \cdot (a - b)\right)$$

T. **Polynomials**:

$$\varpi = \frac{360}{l} = \frac{2 \cdot \pi}{l}$$

ω, angular frequency; it indicates how often the period l is contained in a full rotation of the angle α

$y = cos(\omega t) = sin(\omega t + 90)$
$y = c_0 + c \cdot cos(\omega t - \omega t_0)$

c_0, shifts the curve (+ or -) along the y-axis by c_0

$$sinx = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ...$$

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...$$

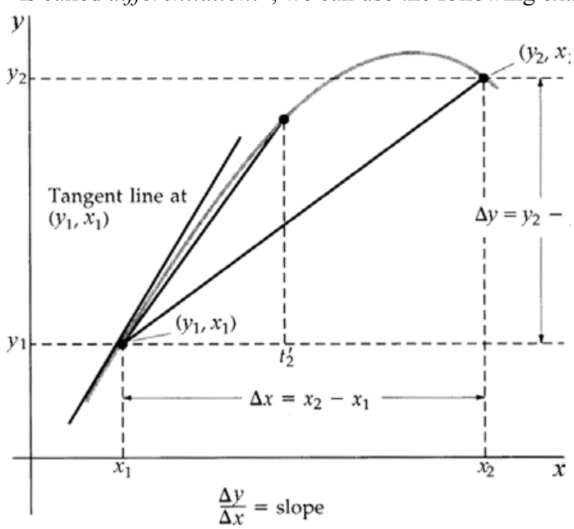
$$tanx = x + \frac{x^3}{3!} + \frac{2 \cdot x^5}{15!} + \frac{3 \cdot x^7}{45!} + ...|x| < \frac{\pi}{2}$$

} x in radians

Differential: in various branches of science, it is sometimes necessary to use the basic tools of calculus, invented by Newton, to describe certain phenomena. Here some basic properties and "rules of thumb" that should be a useful review. First, a function must be specified that relates one variable to another (such as a coordinate as a function of time). Suppose one of the variables is called y (the dependent variable), the other x (the independent variable).

Thus, might have a functional relationship such as: $y(x) = a \cdot x^3 + b \cdot x^2 + c \cdot x + d$

If a, b, c, and d are specified constants, then y can be calculated for any value of x. Differential calculus is based on the notion of the *average rate of change*. The transition from an average rate of change to an instantaneous rate, is the basic idea in differential calculus. This implies that $y = f(x)$ must be a continuos function, that is, those for which y varies "smoothly" with x. At $x = x_1$, the graph of the function must have a *tangent* at the point (x_1, y_1), when both Δx and Δy tend to be 0. The slope of the tangent is also referred to as the *gradient*. Hence, the derivative of y with respect to x is defined as the limit as Δx approaches zero, when the slopes of chords are drawn between two points on the y versus x curve. The operation of finding a derivative of a function is called *differentiation*. , we can use the following example:



When we say that y is a function of x, we mean that for each value of x there is a corresponding value of y; e.g.: $y = A \cdot x^2$, where A is a constant. Graph of a typical function $y = f(x)$. The points (y_1, x_1) and (y_2, x_2) are connected by a straight line. The slope of this line is $\Delta y/\Delta x$. As the interval beginning at x_1 is decreased, the slope for that interval approaches the slope of the line tangent to the curve at x_1, which is the derivative of y with respect to x. Mathematically, we write:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{y(x+\Delta x) - y(x)}{\Delta x}$$

where Δy and Δx are defined as $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$. It is important to note that $\Delta y/\Delta x$ does not mean Δy divided by Δx, but is simply a notation of the limiting process of the derivative as defined with the limes-function.

The derivative of a function of x is another function of x. If y is a constant, the graph of y versus x is a horizontal line with 0 slope. The derivative of a constant is thus 0.

1st derivative:

$y' = \frac{\Delta y}{\Delta x}$ *also known as* $\frac{dy}{dx}$ ...and called **1st prime**.

Δ (delta) is used to indicate any difference

$y = a \cdot x^n$ $\qquad$ $y' = n \cdot a \cdot x^{n-1}$

a useful expression to remember when $y(x) = a \cdot x^n$, where a is a constant and an is any positive or negative integer or fraction.

the slope of the tangent in any point is obtained by the 1st derivative. For:

$x > 0$, we obtain $y' > 0$, all tangents ascend (↑....rising slope)
$x = 0$, we obtain $y' = 0$, no slope at all (vertex) (0....parallel to horizontal x-axis)
$x < 0$, we obtain $y' < 0$, all tangents descend (↓....falling slope)

$y = a$	$\cdots \frac{d}{dx}(a) = 0 \cdots$	$y' =$	0a, constant
$y = x$	$\cdots \frac{d}{dx}(x) = 1 \cdots$	$y' = 1$	
$y = a \cdot x^n$	$\cdots \frac{d}{dx}(a \cdot x^n) = n \cdot a \cdot x^{n-1} \cdots$	$y' = n \cdot a \cdot x^{n-1}$	

Derivative of the sum of two functions: if a function $f(x)$ is equal to the sum of two functions, say, $u(x)$ and $v(x)$, then the derivative of the sum is equal to the sum of the derivatives:

$y = u_{(x)} \pm v_{(x)}$ $\qquad$ $\cdots \frac{d}{dx} f(x) = \frac{d}{dx} \cdot [u(x) \pm v(x)] = \frac{du}{dx} + \frac{dv}{dx} \cdots$ $\qquad$ $y' = u' \pm v'$

Derivative of the product of two functions: if a function *f(x)* is given by the product of two functions, say, *u(x)* and *v(x)*, then the derivative of *f(y)* is defined as:

$y = u_{(x)} \cdot v_{(x)}$ .... $\frac{d}{dx}f(x) = \frac{d}{dx} \cdot [u(x) \pm v(x)] = \frac{du}{dx} + \frac{dv}{dx}$ .... $y' = u' \cdot v + u \cdot v'$

$y = \frac{u_x}{v_x}$ $y' = \frac{u' \cdot v - u \cdot v'}{v^2}$

$y = a^{ex}$ .... $\frac{d}{dx}(e^{ax}) = a \cdot e^{ax}$ .... $y' = a \cdot e^{a \cdot x}$

$y = e^x$ $y' = e^x$

$y = a^x$ $y' = a^x \cdot ln(a)$

$y = ln(a \cdot x)$ .... $\frac{d}{dx}[ln(a \cdot x)] = \frac{1}{x}$ .... $y' = 1/x$

$y = sin(\alpha \cdot x)$ .... $\frac{d}{dx}[sin(\alpha \cdot x)] = \alpha \cdot cos(\alpha \cdot x)$ .... $y' = cos(\alpha \cdot x)$

$y = cos(\alpha \cdot x)$ .... $\frac{d}{dx}[sin(\alpha \cdot x)] = -\alpha \cdot sin(\alpha \cdot x)$ .... $y' = -sin(\alpha \cdot x)$

$y = tan\alpha$ $y' = \frac{1}{cos^2\alpha}$

$y = ctan\alpha$ $y' = -\frac{1}{sin^2\alpha}$

Chain rule of differential calculus: if y = f(x) and x = f(z), then dy/dx can be written as the product of two derivatives:

$y = f(g(x))$ .... $\frac{dy}{dx} = \frac{dy}{dz} + \frac{dz}{dx}$ .... $y' = [f(g(x))]' = f'g(x) \cdot g'(x)$

in compound EQ's, *y'* consists of the product of internal and external derivatives:

$y = f(f(x))$ $y' = f'(x) \cdot x'$

e.g.: $y = sin(\omega \cdot x + \beta)$ e.g.: $y' = \omega \cdot cos(\omega \cdot x + \beta)$

2nd derivative: the second derivative of *y* with respect to *x* is defined as the derivative of the function *dy/dx* (the derivative of the derivative). It is usually written as:

$y = f(x)$ .... $\frac{d''y}{dx''} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ .... $y'' = f''(x)$

A function reaches a *maximum* value at x_0 if $y' = 0$ and $y'' < 0$.

A function reaches a *minimum* value at x_0 if $y' = 0$ and $y'' > 0$.

Point of *inflection* (neither a maximum nor a minimum) with a horizontal tangent: $y'' = 0$

Partial D.: A function with the variable *z* dependent by the independent variables *x* and *y*. The word partial indicates that each independent variable is dealt with separately. The rounded ∂ is therefore used to indicate a partial derivative. Each partial derivative can be treated with the methods mentioned above.

$z = f(x,y)$ $\quad$ $z' = \partial z/\partial x$ $\quad$ *y*, constant

$z' = \partial z/\partial y$ $\quad$ *x*, constant

$$dz = \frac{\partial y}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

P. D. **Equation** (operator):

GRADdient: The gradient (or gradient vector field) of a scalar function *f*(*x*) with respect to a vector variable is denoted by the *nabla* operator (turns a scalar into a vector entity):

$\varphi = f(x,y,z)$ $\quad$ $V_{e1,2,3}$, unity vectors

$$grad\,\varphi = \nabla \cdot f(x,y,z) \qquad grad\,\varphi = \frac{\partial \varphi}{\partial x}V_{e1} + \frac{\partial \varphi}{\partial y}V_{e2} + \frac{\partial \varphi}{\partial z}V_{e3}$$

DIVergence: The divergence of a continuously differentiable vector field is defined to be the scalar-valued function (turns a vector into a scalar entity):

$V_F = f(V_{Fx}, V_{Fy}, V_{Fz})$ $\quad$ $V_{Fx,y,z}$, vectors

$$div V_F = \nabla \cdot F \qquad div V_F = \frac{\partial V_{Fx}}{\partial x} + \frac{\partial V_{Fy}}{\partial y} + \frac{\partial V_{Fz}}{\partial z}$$

ROTation:

In vector calculus, **ROT** is a vector operator that shows a vector field's rate of rotation: the direction of the axis of rotation and the magnitude of the rotation. It can also be described as the **circulation density**. The operator is known as both **rot** or **curl** The spatial distribution of unity vectors in 3D - results in a vector entity:

$V_F = f(V_{Fx}, V_{Fy}, V_{Fz})$ $\quad$ $V_{e1,2,3}$, unity vectors

$rot V_F = \nabla \times F$ $\quad$ $V_{Fx,y,z}$, vectors

$$rot V_F = \left(\frac{\partial V_{Fx}}{\partial y} - \frac{\partial V_{Fy}}{\partial z}\right)V_{e1} + \left(\frac{\partial V_{Fx}}{\partial z} - \frac{\partial V_{Fz}}{\partial x}\right)V_{e2} + \left(\frac{\partial V_{Fy}}{\partial x} - \frac{\partial V_{Fx}}{\partial y}\right)V_{e3}$$

Integral: (Anti-derivative) The limits of a sum (making the whole out of parts).

Integration is related to the problem of finding the area under a curve. It is also the inverse of differentiation.

$f(x)=\dfrac{dy}{dx}=3\cdot a\cdot x^2+b$ which is the result of differentiating the function $y(x) - a\cdot x^3 + b\cdot x + c$

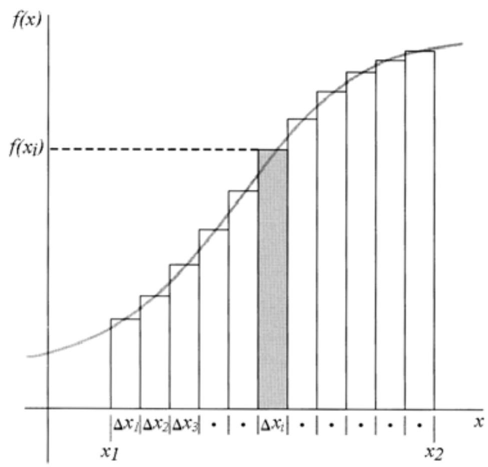
we can write the former equation as $dy = f(x)dx = (3\cdot a\cdot x^2 + b)dx$ and obtain $y(x)$ by 'summing' over all values of x. Mathematically we write this inverse operation:

$y(x)=\int f(x)dx \;=\; \int(3\cdot a\cdot x^2+b)dx$ $\qquad y(x) - a\cdot x^3 + b\cdot x + c$

where c is a constant of the integration. This type of integral is called an indefinite integral because its value depends on the choice of c (not determined). This indefinite integral is written without lower and upper limits. It is in contrast to the definite integral, which is uniquely determined by the upper and lower values (e.g.: x_2, x_1).

A general indefinite integral $I(x)$ is defined as $I(x)=\int f(x)dx$

where $f(x)$ is called the integrand and $f(x)=\dfrac{dI(x)}{x}$



For a general continuos function $f(x)$, the integral can be described as the area under the curve bounded by $f(x)$ and the x-axis, between two specified values of x, say x_1 and x_2.

$$area=\lim_{\Delta x\to 0}\sum f(x_i)\cdot \Delta x_i=\int_{x_1}^{x_2} f(x)dx$$

the shaded area is approximately $f_i\Delta x_i$, where f_i is evaluated anywhere in the interval. This approximation improves if Δx_i is very small. If we sum all these area elements from x_1, to x_2 and take the limit of this sum as Δx_i approaches zero ($\Delta x_i \rightarrow 0$), we obtain the true area under the curve bounded by $f(x)$ and x, between the limits x_1 and x_2. This limit is called the integral of f over x. Integrals of that type are called definite integrals.

One common integral that arises in practical situations has the form

$\int x^n dx=\dfrac{x^{n+1}}{n+1}+c$ $\qquad$ provided n ≠ -1

this result is obvious, being that differentiation of the right-hand side with respect to x gives $f(x) = x^n$ directly. If the limits of the integration are known, this integral becomes a definite integral and is written:

$\int_{x_1}^{x_2} x^n dx=\dfrac{x_2^{\,n+1}-x_1^{\,n+1}}{n+1}$ $\qquad$ provided n ≠ -1

A few rules facilitating integration:

1. The interval of integration may be broken up into a number of subintervals, and the integration performed over each interval separately; thus, breaking (a, b) up into (a, c) and (c, b)
   $$\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$$
2. Interchanging the limits changes the sign of the integrals
   $$\int_a^b f(x)dx=-\int_b^a f(x)dx$$
3. A sum of functions is integrated term by term (with the integrals in/ or definite)
   $$\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx$$
4. A constant factor of the integrand can be put in front of the integral sign
   $\int k\cdot f(x)dx=k\cdot\int f(x)dx$ $\qquad$ e.g.: $\int k\,dx=k\cdot\int dx=k\cdot x$
5. Basic rules of integrations:

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ …. provided n ≠ -1

$\int \frac{1}{x} dx = \int x^{-1} dx = ln(x)$

$\int \frac{1}{a+b\cdot x} dx = \frac{1}{b} ln(a+b\cdot x)$

$\int \frac{1}{(a+b\cdot x)^2} dx = \frac{1}{b\cdot(a+b\cdot x)}$

$\int \frac{1}{(a^2+x^2)} dx = \frac{1}{a}\cdot tan^{-1}\frac{x}{a}$

$\int \frac{1}{(a^2-x^2)} dx = \frac{1}{2\cdot a}\cdot ln\frac{a+x}{a-x}$ …. ($a^2 - x^2 > 0$)

$\int \frac{1}{(x^2-a^2)} dx = \frac{1}{2\cdot a}\cdot ln\frac{x-a}{x+a}$ …. ($x^2 - a^2 > 0$)

$\int \frac{x}{(a^2\pm x^2)} dx = \pm\frac{1}{2}\cdot ln(a^2\pm x^2)$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = sin^{-1}\frac{x}{a} = -cos^{-1}\frac{x}{a}$ …. ($a^2 - x^2 > 0$)

$\int \frac{1}{\sqrt{x^2\pm a^2}} dx = ln\left(x+\sqrt{x^2\pm a^2}\right)$

$\int \frac{x}{\sqrt{a^2-x^2}} dx = -\sqrt{a^2-x^2}$

$\int \frac{x}{\sqrt{x^2\pm a^2}} dx = \sqrt{x^2\pm a^2}$

$\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}\left(x\cdot\sqrt{a^2-x^2} + a^2\cdot sin^{-1}\frac{x}{a}\right)$

$\int x\cdot\sqrt{a^2-x^2}\, dx = \frac{1}{3}(a^2-x^2)^{3/2}$

$\int x\cdot\sqrt{x^2\pm a^2}\, dx = \frac{1}{3}(x^2-a^2)^{3/2}$

$\int \sqrt{x^2\pm a^2}\, dx = \frac{1}{2}\left[x\cdot\sqrt{x^2\pm a^2} \pm a^2\cdot ln\left(x+\sqrt{x^2\pm a^2}\right)\right]$

$\int \frac{1}{(x^2+a^2)^{3/2}} dx = \frac{x}{a^2\cdot\sqrt{x^2+a^2}}$

$\int \frac{x}{(x^2+a^2)^{3/2}} dx = -\frac{1}{\sqrt{x^2+a^2}}$

$\int a^x dx = \frac{a^x}{ln(a)}$

$\int e^x dx = \frac{e^x}{ln(e)} = e^x$

$\int e^{a\cdot x} dx = \frac{1}{a}\cdot e^x$

$\int ln(a^x) dx = x\cdot ln(a\cdot x) - x$

$\int x\cdot e^{a\cdot x} dx = \frac{e^{a\cdot x}}{a^2}\cdot(a\cdot x - 1)$

$\int \frac{1}{a+b\cdot e^{c\cdot x}} dx = \frac{x}{a} - \frac{1}{a\cdot c} ln\left(a+b\cdot e^{c\cdot x}\right)$

for simplicity, C is omitted in the following steps

$\int sin(x) dx = -cos(x)$

$\int sin(a\cdot x) dx = -\frac{1}{a} cos(a\cdot x)$

$\int sin^2(a\cdot x) dx = \frac{x}{2} - \frac{sin(2\cdot a\cdot x)}{4\cdot a}$

$\int sin^{-1}(a\cdot x) dx = x\cdot sin^{-1}(a\cdot x) + \frac{\sqrt{1-a^2\cdot x^2}}{a}$

$\int \frac{1}{sin^2(a\cdot x)} dx = -\frac{1}{a} ctan(a\cdot x)$

$\int cos(x) dx = sin(x)$

$\int cos(a\cdot x) dx = \frac{1}{a} sin(a\cdot x)$

$\int cos^2(a\cdot x) dx = \frac{x}{2} + \frac{sin(2\cdot a\cdot x)}{4\cdot a}$

$\int cos^{-1}(a\cdot x) dx = x\cdot cos^{-1}(a\cdot x) - \frac{\sqrt{1-a^2\cdot x^2}}{a}$

$\int \frac{1}{cos^2(a\cdot x)} dx = \frac{1}{a} tan(a\cdot x)$

$\int tan(x) dx = \int \frac{sin(x)}{cos(x)} = -ln[cos(x)]$

$\int tan(a\cdot x) dx = -\frac{1}{a}\cdot ln[cos(a\cdot x)]$

$\int tan^2(a\cdot x) dx = \frac{1}{a} tan(a\cdot x) - x$

$\int ctan(x) dx = \int \frac{cos(x)}{sin(x)} = -ln[sin(x)]$

$\int ctan(a\cdot x) dx = \frac{1}{a} ln[sin(a\cdot x)]$

$\int ctan^2(a\cdot x) dx = -\frac{1}{a} ctan(a\cdot x) - x$

6. Three ways of solving an integral $x = f(t)$:
   a) integration with the rules stated above
   b) splitting variables: $\frac{dx}{dt} = -k \cdot x$

   rearranging equation: $\frac{dx}{x} = -k \cdot dt$

   integrating equation: $\int_{x=0}^{x} \frac{1}{x} dx = -k \int_{t=0}^{t} \frac{1}{dt}$

   resolving integral: $ln(x)\big|_0^x + c_1 = -k \cdot t\big|_0^t + c_2$

   executing (upper - lower) limits $ln(x) - ln(x_0) + c_1 = -k \cdot t\big|_0^t - 0 + c_2$

   rearranging equation: $ln\left(\frac{x}{x_0}\right) = -k \cdot t - c_1 + c_2$ $c = - c_1 + c_2$

   executing antilog: $\frac{x}{x_0} = e^{-k \cdot t} \cdot e^c$

   rearranging equation: $x = x_0 \cdot e^{-k \cdot t + c}$
   c) trail and error: using a set of real numbers and approximate results according to the numbers tried.

Partial Integration: Sometimes it is useful to apply the method of partial integration (also called "integrating by parts") to evaluate certain integrals. The method uses the property that:

$$\int u\,dv = u \cdot v \int v\,du$$

where u and v are carefully chosen so as to reduce a complex integral to a simpler one. In many cases, several reductions have to be made. Consider the function:

$$I(x) = \int x^2 \cdot e^x dx$$

this can be evaluated by integrating by parts twice. First, if we choose $u = x^2$, $v = e^x$, we get:

$$\int x^2 \cdot e^x dx = \int x^2 \cdot d\left(e^x\right) = x^2 \cdot e^x - 2\int x \cdot e^x dx$$

now, in the second term, choose $u = v$, $v = e^x$, which gives:

$$\int x^2 \cdot e^x dx = x^2 \cdot e^x - 2 \cdot x \cdot e^x + 2\int e^x dx + c_1$$

$$\int x^2 \cdot e^x dx = x^2 \cdot e^x - 2 \cdot x \cdot e^x + 2 \cdot e^x + c_2$$

The perfect Integral: another useful method to remember is the use of the perfect differential, in which we look for a change of variable such that the differential of the function is the differential of the independent variable appearing in the integrand. For example, consider the integral:

$$I(x) = \int cos^2(x) \cdot sin(x) dx$$

This becomes easy to evaluate if we rewrite the differential as $d(cosx) = -sin(x)dx$. The integral then becomes:

$$\int cos^2(x) \cdot sin(x) dx = -\int cos^2(x) \cdot d(cos(x))$$

If we now change variables, letting $y = cos(x)$, we get:

$$\int cos^2(x) \cdot sin(x) dx = -\int y^2 dy = -\frac{y^3}{3} + c = -\frac{cos^3 x}{3} + c$$

Scalar: Undirected quantity; a scalar does not have a direction but is associated with a magnitude; mass, temperature, density, etc. are scalars.

Vector: A vector (directed quantity) is denoted with a quantitative value (length) pointing in a certain direction (angle), represented by an arrow. *X* and *y* are the components or coordinates of a vector. Force, velocity, acceleration, etc. are vectors.

absolute value and angle of a vector:	$\|a\| = \sqrt{a_x^2 + a_y^2}$ *)	*) within a 2-dimensional system
	$\alpha = tan \frac{a_x}{a_y}$	***a***, vector, where | ***a*** | the absolute amount
components of a vector:	$a_x = \boldsymbol{a} \cdot cos\alpha$ $a_y = \boldsymbol{a} \cdot sin\alpha$	$a = \begin{pmatrix} a_x \\ a_y \end{pmatrix}$ a_x, a_y are the components of the vector ***a***
Collinear:	parallel to each other	
Coplanar:	2D-plane parallel to another	
Orthogonal:	perpendicular to each other	
Addition:	$a + b = \sqrt{\Sigma_x^2 + \Sigma_y^2}$	
Subtraction:	$a - b = \sqrt{\Sigma_x^2 - \Sigma_y^2}$	
Multiplication:	$a \cdot b = a_x \cdot b_x + a_y \cdot b_y$ = scalar i.e.: $cos\alpha = 0$	Multiplying two vectors results in a scalar
	$a \cdot k = a_x \cdot k + a_y \cdot k$	Multiplication of a vector with a constant (scalar) yields an other vector different in length; if k is positive (>0) vector points in the same direction; if negative (<0) vector pints into the opposite direction.
	$a \times b = \|a\| \cdot \|b\| \cdot cos(\alpha)$ = vector $a, b \neq 0$ i.e.: $cos\alpha = 90°$	the inner product of two vectors ***a*** and ***b*** is equal to the product of three factors: the absolute amounts of ***a**, **b***, and the cosine of the angle between the two vectors; this implies that the two vectors are perpendicular (orthogonal) to each other.
Division:	$\frac{a}{b} = a_x \cdot \frac{1}{b_x} + a_y \cdot \frac{1}{b_y}$	

Probability: For a detailed introduction see E. Batschelet *Introduction to Mathematics for Life Science*, Springer Verlag, Heidelberg

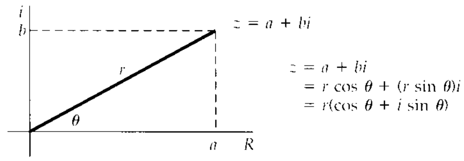
True value	$\bar{x} = \lim_{n \to \infty} x$	theoretical
Arithmetic mean	$\bar{x}_{AM} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$	quantitative ratio of an average value; usually greater or equal to the geometric mean;
Geometric mean	$\bar{x}_{GM} = \sqrt[n]{x_1 \cdot x_2 \cdot x_3 \cdot .... \cdot x_n}$	divides the area enclosed by the graph into two equal halves;
Poisson Distribution:	$f(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$	μ, arithmetic mean (1st moment) !, factorial, x = 1·2·3·.....·x; x = integer when objects are randomly dispersed or randomly distributed over a region generally follow a Poisson distribution, with x (samples) hardly exceeding values over 10 or 20.
Gauss Distribution:	$f(x) = \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma} \cdot e^{-\frac{(x-\mu)^2}{2 \cdot \sigma^2}}$	σ ≈ Δx, standard deviation* σ^2, variance; μ ≈ $\bar{\mathbf{x}}$, mean value. The *normal* distribution with mean zero and standard deviation one. Generally a bell-shaped and fairly symmetric (with respect to the mean) graph, where x is very large.
Log Norm Distribution:	$f(x) = \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma} \cdot e^{-\frac{(ln(x)-\mu)^2}{2 \cdot \sigma^2}}$	as a non-logarithmic (ln) chart reflects Poisson characteristics, but with the abscissa (ln*x*) converts into Gauss distribution; μ ≈ $\bar{\mathbf{x}}$, mean value.

Deviation.: The absolute value can never be obtained, since it does not provide an exact measurement, therefore the relative error is commonly used which is given as a percentage value [%].

Mean deviation of single value	$\Delta x_s = \pm \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{(n-1)}}$	equals standard deviation σ
Mean deviation of mean value	$\Delta x_s = \pm \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n \cdot (n-1)}} = \pm \frac{\Delta x}{\sqrt{n}}$	the expression within the prentices can take both positive as well as negative values, therefore the sum of the differences equals 0 once n → ∞
Absolute deviation:	$F_{abs} = \bar{x} \pm \Delta x$	entirely given with the appropriate unit of $\bar{\mathbf{x}}$ (not dimensionless);
Relative deviation:	$f_{abs} = \bar{x} \pm \frac{\Delta x}{\bar{x}} \cdot 100$	with $\bar{\mathbf{x}}$ having its unit, whereas the deviation is given as a %-value
(*) Standard deviation: (Gauss' propagation of deviation)	$\sigma = \sqrt{\Sigma \left(\frac{\partial f \cdot \sigma_i}{\partial x_i} \right)^2}$	$x_1, x_2, x_3 .... = x_i$ $\sigma_1, \sigma_2, \sigma_3 .... = \sigma_i$,

Complex Number: A general complex number *z* can be written as *z = a + bi* where *a* and *b* are real numbers and *i =* √*(-1)*. The quantity *a* is called the real part and the quantity *ib* is called the imaginary part of *z*. Whit *z =* √*(-1)*, it is not possible to state that the expression √*(-1)* is greater or smaller than a given real number. Therefore it is necessary to attribute numbers to have a size with a specific order. √*(-1)* can not be called a number, but an imaginary number with the notation *i =* √*(-1)*. Imaginary numbers can be added, subtracted, multiplied or divided, together with the real numbers can form a set of complex numbers.

Complex Plane: The real number line is the horizontal axis and is denoted by x. The imaginary number is denoted by *y* and is the vertical axis. The imaginary number ***i*** *(0,1)* in the ordinary x-y coordinate system is known as the imaginary unit.



Representation of a complex number in a plane. The real part of the complex number is plotted along the horizontal axis, and the imaginary part is plotted along the vertical axis.

$|Z| = r$

$Z = a + i \cdot b$

$Z = r \cdot (\cos\varphi + i \cdot \sin\varphi) = Z \cdot e^{i \cdot \varphi}$

a, b, real numbers
i, imaginary number = √(-1)
r, magnitude

$r = \sqrt{a^2 + b^2}$

$\tan\varphi = a/b$

$a = r \cdot \cos\varphi$

$b = r \cdot \sin\varphi$

φ, argument, angle between the components *a*, and *b*

$e^{i \cdot \varphi} = \cos\varphi + i \cdot \sin\varphi$

$e^{-i \cdot \varphi} = \cos\varphi - i \cdot \sin\varphi$

Euler's equation

Matrix: Matrices are rectangular arrays of numbers.

Determinants A symbol with vertical bars (not a matrix anymore) with the quantity $a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

Quadratic matrix: Equal matrices must have the same number of rows and columns. Unequal matrices can be extended with the unit-row (u_x u_y u_z), each holding the value of 1, to become quadratic.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{32} & a_{32} & a_{33} \end{pmatrix}$$

$$D_{a_{11}} = a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{12} \cdot \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} + a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Coordinate system: Graphical methods to display certain patterns, usually functions or distributions on a 2-dimensional scale. Some of the listed systems are especially adapted to suite particular problems.

2D - Cartesian S.: The preferred system to describe the relationship of a dependent *y* with an independent variable *x*.

P defined by distances on the x- and y- axis

3D - Cartesian S.: Spatial chart to describe the relationship of a dependent *z* with two independent variables *x* and *y*.

P defined by distances on the x- and y- and z- axis

Polar S.: Used to display vectors, complex numbers, and other relationships involving directed amounts with their angle of attack.

$r = \sqrt{a^2 + b^2}$	*r*, amount
$x = r \cdot cos\varphi$	*x*, equivalent of r on x-axis
$y = r \cdot sin\varphi$	*y*, equivalent of r on y-axis

Cylindrical S.: In physics, where only *z* is the dependent variable, and *x* equals *y*; e.g.: to investigate laminar flow in a tube, reducing the 2D relationship to a single variable *z*.

$Z = Z$	*Z*, amount
$x = Z \cdot cos\varphi$	*x*, equivalent of Z on x-axis
$y = Z \cdot sin\varphi$	*y*, equivalent of Z on y-axis

Spherical S.: In physics, when isotropic factors, like a spherical distribution; e.g.: a single directed variable (vector) is displayed with its angles describing it in space.

$r = r$	*r*, amount
$\varphi = r$	φ, angle of x-axis
$\upsilon = r$	υ, angle of y-axis